

# Gambling on Innovation with Learning

Darrell Velegol\*



Cite This: *Ind. Eng. Chem. Res.* 2022, 61, 18457–18463



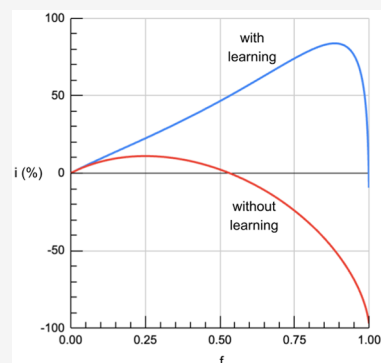
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**ABSTRACT:** To maximize your median return ( $i$ ) and minimize your probability of “going bust” in innovation, you need to invest the proper amount in your innovation projects. A previous article (*IECR* 2021; DOI 10.1021/acs.iecr.1c00511) showed that the Kelly gambling (or investing) strategy helps to make the optimal decisions. To use the Kelly strategy, you must know the anticipated win ratio ( $b$ ) of an innovation project and its estimated probability ( $p$ ) of success. But both  $b$  and  $p$  will likely increase when you bet a fraction ( $f$ ) of your innovation budget on a project, due to learning that increases both  $b$  and  $p$ . This article analyzes how the optimal  $f$  changes when learning takes place. I use simple linear approximations  $p = p_0 + \pi f$  and  $b = b_0 + \beta f$ . As  $\pi$  and/or  $\beta$  increases, the optimal Kelly Criterion value  $f_{KC}$  increases, often significantly, calling for a higher resource investment, and also giving a dramatic increase in growth rate ( $i$ ). It is proposed that organizations measure  $a$ ,  $b_0$ ,  $p_0$ ,  $\pi$ ,  $\beta$ , and  $k$ , where  $k$  is the rate of attempts made per time, and focus on improving them. This article provides the framework to use these parameters, enabling probability processing to improve the probability of success.



## 1. INTRODUCTION. THE QUESTION IN THIS ARTICLE

The purpose of this article is to provide an “algorithm for innovation” for budgeting the fraction or amount of resources to invest in projects of an innovation portfolio, when *learning* is gained during the course of projects. Compared with an earlier article,<sup>1</sup> the primary change in the conditions is that, instead of a fixed win ratio ( $b$ ) and probability of success ( $p$ ) that are *independent* of bet size, these values *change* with bet size, since presumably investing a fraction ( $f$ ) of your resources will lead to learning that increases  $b$  and  $p$ . And so given a set of innovation projects, with estimated win ratios ( $b$ 's), probabilities of success ( $p$ 's), and learning coefficients (defined later in eqs 7a and 7b), my question is “What fraction to invest in each project, when learning is taken into account, in order to maximize your growth rate?” That is, how should innovation leaders gamble on their innovation projects, given learning that can happen when they invest? Figures 1–3 give the key results.

The allocation of innovation resources is an essential decision for chemical research to reach fruition and is part of the process design for innovation processes. There are numerous reasons to bet according to the Kelly Criterion (KC), and the rationale for these was discussed in an earlier article.<sup>1</sup> In short, other allocation methods do not guide what fraction ( $f$ ) of resources to bet on each project in your innovation portfolio, and often lead to “going bust”, although some companies might choose to continue to invest in suboptimal or even poor innovation processes, sometimes without realizing it.

In a previous article, I showed that given estimated paybacks ( $b$ ) of projects in an innovation portfolio, and estimated probabilities ( $p$ ) of success, that you can predict the optimum

fraction ( $f_{KC}$ ) of your organization's resources to bet on each project, to maximize growth.<sup>1</sup> The core concept behind that article was the use of the Kelly Criterion (KC). The KC maximizes the logarithm of the current wealth ( $W$ ) for a sequence of bets or investments.<sup>2</sup> Equivalently, the KC maximizes the *geometric* mean of a sequence of bets, rather than the *arithmetic* mean.

In this article, I examine the role of learning. Whereas previously I assumed fixed values for  $b$  and  $p$ , here I presume that if I invest resources in a project, that I will increase  $p$  and/or  $b$ . A higher  $b$  and  $p$  lead to a higher  $f$ , and so the question I ask is what is the optimum value ( $f_{KC}$ ), given a rate of learning that leads to increasing  $b$  and  $p$ ?

## 2. BRIEF REVIEW OF KELLY CRITERION WITH NO LEARNING

Let me first briefly review the previous theory. The ratio ( $\mathfrak{R}$ ) of current wealth ( $W$ ) to initial wealth ( $W_0$ ) depends on (1) the numbers of wins (small  $w$ ) and losses ( $L$ ) that occur, (2) the loss ratio ( $a = \text{loss}/\text{investment}$ ) and win ratio ( $b = \text{profit}/\text{investment}$ , sometimes called the “payback odds”), and (3) the fraction ( $f$ ) of your resources that you bet on the innovation. Following Kelly, I'll analyze the geometric mean of wealth. The geometric

**Received:** September 21, 2022

**Revised:** November 21, 2022

**Accepted:** November 28, 2022

**Published:** December 12, 2022



mean is always less than or equal to the arithmetic mean for a series of numbers, which can have devastating risk consequences for unaware gamblers or investors. The resulting  $\mathfrak{R}$  is given by

$$\mathfrak{R} = \frac{W}{W_0} = (1 + fb)^w(1 - fa)^L \quad (1)$$

where the first term describes the gain from winning, and the second term describes the loss from losing. Thus, the equation accounts for both reward and risk. For a sufficiently large number of plays ( $n$ ),  $w \approx pn$  and  $L \approx qn$ , where  $q = 1 - p$  is the probability of losing. For a constant rate ( $k [=]$  projects/time) of plays, the number ( $n$ ) of plays is given by

$$n = kt \quad (2)$$

where  $t$  is time. Note that we are using binary (win–lose) games here. A future article will assess the role of multiple outcomes or even continuous distributions of winning and losing. Thus, I write eq 1 as

$$\mathfrak{R} = \frac{W}{W_0} = (1 + fb)^{ktp}(1 - fa)^{kt(1-p)} \quad (3)$$

In using eq 3, I'm not using the "expectation value" (i.e., arithmetic mean), which does not fully account for the bet-by-bet risk that I take. Rather, eq 3 gives a distribution of the outcomes. I have often heard my industrial colleagues say how their company will not accept innovation investments that promise less than a 4:1 or 10:1 or higher payback odds, and yet their companies grow at less than 20% (or even less than 5%) per year. One reason is that the (unspoken) losing projects are offsetting the winning projects, giving a lower net growth rate.

In order to maximize  $\mathfrak{R}$ , I set  $d\mathfrak{R}/df = 0$  in the usual way. It turns out to be easier to work with the logarithm of  $\mathfrak{R}$  than  $\mathfrak{R}$  itself, and since  $\ln \mathfrak{R}$  is monotonic, if we maximize  $\ln \mathfrak{R}$ , then we also maximize  $\mathfrak{R}$ .

$$\ln \mathfrak{R} = pkt \ln(1 + fb) + (1 - p)kt \ln(1 - fa) \quad (4)$$

$$\frac{d \ln \mathfrak{R}}{df} = kt \left[ \frac{pb}{1 + fb} - \frac{(1 - p)a}{1 - fa} \right] \quad (5)$$

Setting this equal to zero and solving for  $f$  gives the well-known Kelly Criterion for the fraction of resources one should play on a given bet:

$$f_{\text{KC}} = \frac{p}{a} - \frac{1 - p}{b} \quad (6)$$

Betting the Kelly fraction ( $f_{\text{KC}}$ ) yields the maximum growth rate for a given set of  $\{a, b, p\}$  parameters. The ramifications of eq 6 in terms of growth rate and ruin rate are detailed in ref 1.

For favorable bets, one finds that when the investment fraction ( $f$ ) is small, that the return  $\mathfrak{R} > 1$ , and rises with increasing  $f$ . This is seen in later figures in this article. However, this rise continues only up to a maximum point, which is given by eq 6. Beyond the maximum point, the  $\mathfrak{R}$  decreases with  $f$ , and eventually  $\mathfrak{R}$  falls below unity. In short, the bettor put "too many eggs in one basket".

### 3. KELLY CRITERION WITH LEARNING

The important change in this article is the incorporation of learning and its impact on  $f_{\text{KC}}$ . If a company has a diverse Integrated Innovation Team,<sup>3</sup> an important role of that team is using the company's innovation investment to increase the probability ( $p$ ) of commercial success, and to increase the win

ratio ( $b = \text{profit}/\text{investment}$ ) of each project. That is, the learning that the Team attains will increase  $p$  and  $b$ . I will incorporate the learning into the Kelly algorithm using simple linear functions:<sup>4</sup>

$$p = p_0 + \pi f \quad (7a)$$

$$b = b_0 + \beta f \quad (7b)$$

If the learning follows more sophisticated functions, I could use these, or I could linearize these functions around  $p_0$  and  $b_0$ , without changing the concepts from this article. Furthermore, since  $f = E/W_0$ , all the results to follow can be translated from fraction of resources spent ( $f$ ) to actual expenditure ( $E$ ), as long as the initial capital ( $W_0$ ) is known.

What happens when we have learning, and  $\pi$  and  $\beta$  are no longer zero? The value of  $f_{\text{KC}}$  increases, but it turns out that (1) exact analytical solutions like eq 6 do not exist, and (2) approximate (linearized) analytical solutions are large and messy, making them quite difficult to use compared with eq 6. So in this article, I'll use a Google Spreadsheet (given at <https://tinyurl.com/mr3vtnyk>), which calculates the new  $f_{\text{KC}}$  results quickly and easily to three decimals using "brute force", and I'll show representative examples (e.g., Figures 1–3). The sheet checks all  $f$ 's from 0 to 1 in increments of 0.001, uses eqs 7a and 7b to calculate  $p$  and  $b$ , and from the list of  $\ln \mathfrak{R}$  as a function of  $f$ , the sheet finds the optimal  $\mathfrak{R}$  and  $f_{\text{KC}}$ .

To use the sheet you'll need to make a copy for yourself so you have "Edit" permission. In the "KC" tab of the spreadsheet, there is a green region where you can enter your values for  $a$ ,  $b_0$ ,  $\beta$ ,  $p_0$ , and  $\pi$ . The sheet then calculates the optimal Kelly fraction ( $f_{\text{KC}}$ ). The calculation is more sensitive to changes in probability (due to  $\pi$ ) than to changes in the win ratio (due to  $\beta$ ). As expected, if one overestimates  $\beta$  or  $\pi$ , the value of  $f_{\text{KC}}$  comes out larger than reality, and vice versa. For small mistakes the error in  $f_{\text{KC}}$  is symmetric with the error in  $\beta$  or  $\pi$ , although for larger uncertainties, the numerical calculation is needed.

Once I numerically solve for  $f_{\text{KC}}$  from the spreadsheet, I calculate the growth rate ( $g$ ) using this value of  $f$ . However,  $i$  is "per instance", and  $n = kt$ , and so to calculate  $g$  in %/yr (or more generally, % per some time  $t_0$ ) requires just a bit of thought.

$$\mathfrak{R} = (1 + i)^n = (1 + g)^{t/t_0} \quad (8)$$

$$\begin{aligned} \ln \mathfrak{R} &= n \ln(1 + i) \\ &= \frac{t}{t_0} \ln(1 + g) \\ &= pn \ln(1 + fb) + qn \ln(1 - fa) \end{aligned} \quad (9)$$

$$\begin{aligned} \frac{t}{nt_0} \ln(1 + g) &= p \ln(1 + fb) + q \ln(1 - fa) \\ &= \frac{1}{kt_0} \ln(1 + g) \end{aligned} \quad (10)$$

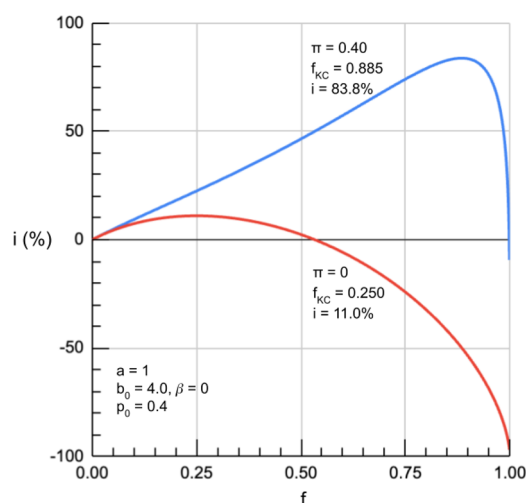
For  $g < 23\%$ ,  $\ln(1 + g) \approx g$  with good accuracy, so eq 15 becomes

$$g \approx kt_0 [p \ln(1 + fb) + q \ln(1 - fa)] = kt_0 i \quad (11)$$

Thus I can go between  $g$  and  $i$ . Equation 11 tells us the mathematical steps required to increase the growth rate ( $g$ , in %/time), which might seem obvious, but now we see precisely how each factor contributes:

- Increase  $k$ . If you work through projects more quickly,  $g$  increases. Fail faster.
- Increase  $p$ . If you increase the probability of success,  $g$  increases. Learn and so increase  $p$ .
- Increase  $b$ . If you increase the value of the project,  $g$  increases. Learn and so increase  $b$ .
- Decrease  $a$ . If you mitigate the loss of the project,  $g$  increases.
- Optimize  $f$ . For any combination of parameters, there is an optimal  $f$ , which I find in this article. For the simple case with no learning, I recover the Kelly Criterion (KC) in eq 6.

Figures 1 through 3 provide three examples that show the importance of learning. Figure 1 shows the result when  $p$  grows

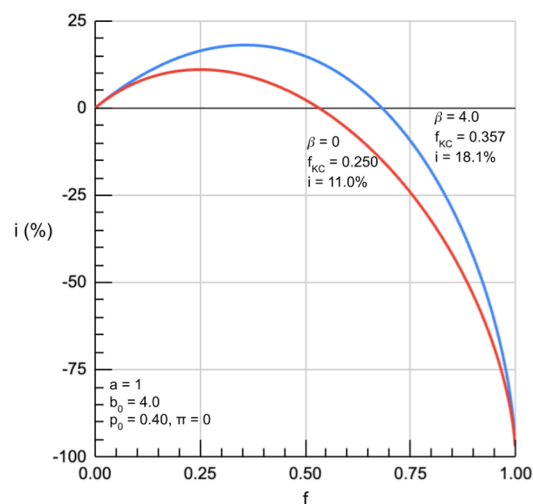


**Figure 1.** Increase in  $f_{\text{KC}}$  and median  $i$  with learning. Here I set a finite value of  $\pi = 0.40$  (i.e., probability growth with  $f$ ), and I set  $\beta = 0$ . The figure shows  $i$  for various bet fractions ( $f$ ) from 0 to 1.00. In this figure  $a = 1$ ,  $b_0 = 4$ ,  $\beta = 0$ , and  $p_0 = 0.4$ . The red (lower) curve is for when  $\pi = 0$ , and the blue (upper) curve is for  $\pi = 0.40$ . Both show a maximum  $i$  at a particular  $f$ . When  $\pi = 0$ , the maximum  $i$  occurs at  $f = 0.250$ , as predicted by the KC from eq 6. When learning (i.e., here,  $\pi = 0.40$ ) occurs, the value of the maximum  $f$  increases, here to 0.885. The  $i$  increases from 11.0% to 83.8%.

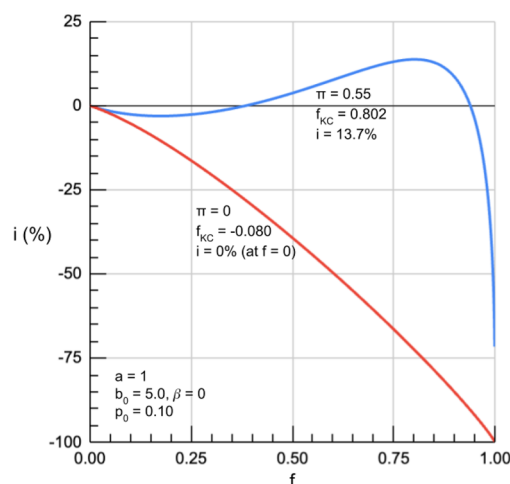
with investment fraction ( $f$ ), and Figure 2 shows the result when  $b$  grows with  $f$ . All the  $\mathcal{R}$  values are median values (50th percentile). To see how to calculate other percentiles, see section 4. In each of Figures 1 to 3, we see a maximum growth rate ( $i$ ) at some value of  $f = f_{\text{KC}}$ . Learning causes this value to move to the right (higher  $f_{\text{KC}}$ ), also giving a higher value of growth rate ( $i$ ). We see that if we move too far to the right, that  $i$  falls below 0 (i.e., loss). When this happens, it is important to ignore sunk costs and to exit the project quickly to minimize the loss.

In Figure 3 we see a curious dip at  $f = 0.175$ , with a corresponding negative  $i$ . This is a case of “go big or go home”, and I suspect that this is the more usual case, that many innovation projects will actually lose money, until a sufficient amount is invested to make the project a winner, as we see toward the right side of Figure 3. Knowing this at the start of the project can mitigate a potential discouragement to the innovation team, including leadership!

One more important point is that the spreadsheet calculator is for single projects. It will also apply to a set of projects, as long as  $\sum f_i < 1$ . As seen in ref 1, if for all projects we have  $\sum f_i > 1$ , then we



**Figure 2.** Increase in  $f_{\text{KC}}$  and median  $i$  with learning. Here I set a finite value for  $\beta = 4.0$ , and I set  $\pi = 0$ . The figure shows  $i$  for various bet fractions ( $f$ ) from 0 to 1.00. In this figure  $a = 1$ ,  $b_0 = 4$ ,  $\pi = 0$ , and  $p_0 = 0.4$ . The red (lower) curve is for when  $\beta = 0$ , and the blue (upper) curve is for  $\beta = 4.0$ . Both show a maximum  $i$  with  $f$ . When  $\beta = 0$ , the maximum  $i$  occurs at  $f = 0.250$ , as predicted by the KC from eq 6 (same as Figure 1). When learning (i.e.,  $\beta = 4.0$ ) occurs, the value of the maximum  $f$  increases to 0.357. The  $i$  increases from 11.0% to 18.1%.



**Figure 3.** Project becoming profitable due to learning. In this figure  $a = 1$ ,  $b_0 = 5.0$ , and  $p_0 = 0.10$ . The red curve shows the result before learning, a monotonically downward curve with  $f = -0.08$  (i.e., avoid this bet since  $i < 0$  for any finite bet size). With learning (blue curve with  $\pi = 0.55$ ,  $\beta = 0$ ) the curve changes to profitable. Significantly, for a small investment ( $f = 0.175$ ),  $i = -3.08\%$ ; thus  $i < 0$ , and this would be a losing investment! But as more is invested (higher  $f$ ), the  $i$  becomes positive, reaching a max of  $i = 13.7\%$  at  $f = 0.802$ . If your organization has a history of significant  $\pi$ -type learning, then you can turn unprofitable ventures into profitable ones.

need to do a deeper numerical calculation that has a constraint of  $\sum f_i = 1$ . This calculation is not contained in this article.

As stated earlier, I might conclude based on these ideas that the role of a company’s innovation team (i.e., the entire team, including R&D, as well as commercial leadership, marketing, manufacturing, legal, finance, regulatory, supply chain, and other functions) is to produce a high value of  $\pi$  and  $\beta$  to increase  $p$  and  $b$ . As work progresses, this is done in part because failures (i.e., hypotheses or alternatives that are false or implausible) are “removed from the marble pot”. Similarly, you might imagine a

roulette wheel that goes from having 38 numbers down to 34 or 28 perhaps. Your probability of winning increases.

#### 4. RUIN AND OTHER QUANTILES

As eq 1 reveals, there is a distribution of potential outcomes for any given values of  $a$ ,  $b$ ,  $p$ , and  $f$ . Figures 1 to 3 show the median values, but I might be interested in knowing other quantiles of outcomes, including “ruin”. Wins and losses can come in any order, and so I need to know how many ways a given number of wins and losses can occur. The frequency ( $g$ ) of any outcome of wins and losses for eq 1 is given by a Bernoulli distribution:

$$g(n, w, p) = \frac{n!}{w!(n-w)!} p^w (1-p)^{n-w} \quad (12)$$

where  $n = kt$  at any time ( $t$ ). For  $np > 5$  and  $n(1-p) > 5$ , the binomial distribution can be closely approximated by a normal distribution, and so I'll let  $x$  be a continuous variable that replaces the integer value  $w$ :

$$g(x) = \frac{1}{\sqrt{2\pi s^2}} \exp\left(-\frac{(x-m)^2}{2s^2}\right)$$

$$m = pn$$

$$s = \sqrt{np(1-p)} \quad (13)$$

I now have a continuous form that closely approximates the full distribution of outcomes. For given values of  $a$ ,  $b$ ,  $f$ , and  $p$ , I can solve for any quartile, percentile, or other quantile. In Excel or Google Sheets, the command to find the Cumulative Distribution Function (CDF) is

$$=NORM.DIST(y, m, s, TRUE)$$

Plugging in  $y = 0$  for a normal distribution  $N(0,1)$  (i.e., mean = 0 and standard deviation = 1) produces a value of 0.50 (median). A  $y$  of  $-0.675$  gives 0.25 (25%, or first quartile), while a  $y$  of  $+0.675$  gives 0.75 (75%, or third quartile). To be explicit, for this last calculation for the third quartile, I would type =NORM.DIST(0.675,0,1,TRUE) to get 0.750.

Let us start by calculating where “ruin” occurs. In my previous article (ref 1), I defined ruin as  $\mathfrak{R}_{\text{ruin}} = 0.01$ . I'll write eq 1 letting the continuous variable  $x$  replace  $w$ :

$$\mathfrak{R} = \frac{W}{W_0} = (1+fb)^x (1-fa)^{(n-x)} \quad (14)$$

I solve eq 14 for  $x$ , for any values  $a$ ,  $b$ ,  $f$ , and  $\mathfrak{R}_{\text{ruin}}$ , to give

$$x = \frac{\ln \mathfrak{R}_{\text{ruin}} - n \ln(1-fa)}{\ln(1+fb) - \ln(1-fa)} \quad (15)$$

Equation 15 finds the number of wins ( $x$ , which here is a continuous real variable, rather than the integer  $w$ ) required to reach a given value of  $\mathfrak{R}_{\text{ruin}}$ , and I therefore can calculate a value  $x_{\text{ruin}}$ . Now I can immediately find the fraction of cases in which my parameters will lead to ruin, by taking the CDF:

$$\text{CDF}_{\text{ruin}} = \frac{1}{2} \left[ 1 + \text{erf}\left(\frac{x_{\text{ruin}} - m}{\sqrt{2s^2}}\right) \right] = \frac{1}{2} \left[ 1 + \text{erf}\left(\frac{y}{\sqrt{2}}\right) \right] \quad (16)$$

where

$$x = ys + m \quad (17)$$

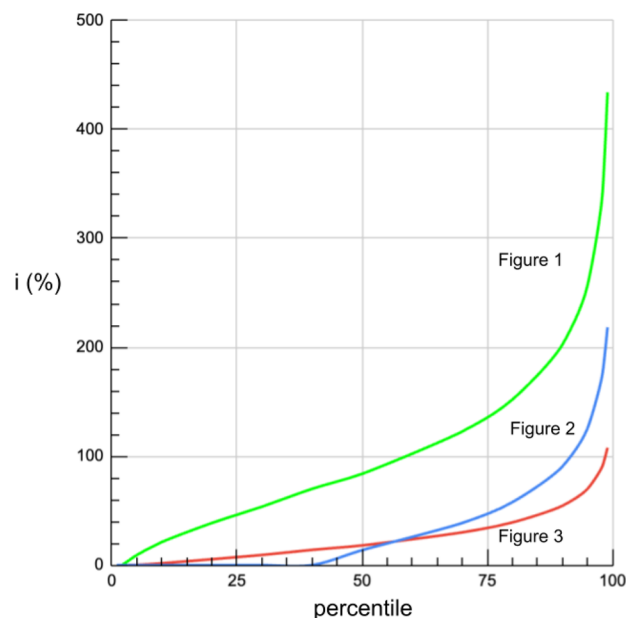
Equivalently,  $y = (x - m)/s$ . This CDF calculation gives the ruin rate for a given set of parameters.

Now I'll engage the second problem and calculate the outcome for any quantile I want. As described above, if I want the first decile value (i.e., 10th percentile), I find the  $y$  for CDF = 0.10. The command for this inverse CDF function in Excel is

$$=NORMINV(\text{probability}, m, s)$$

where for the 10th percentile I would set probability = 0.10,  $m = 0$ , and  $s = 1$ . Then I calculate  $x$  from eq 17, and put into eq 14 for  $\mathfrak{R}$ . I can then get the growth rate ( $i$  or  $g$ ) using eq 8.

In Figure 4 I show the growth rate ( $i$ ) for various percentile cases. The 50th percentile is the median case, which is given in



**Figure 4.** Growth rates ( $i$ ) from the examples in Figures 1 to 3 at various percentiles. Here  $i$  is plotted at various percentiles of outcomes (50th is median). I often use the 10th percentile result for a “poor” case, and a 90th percentile result for a “good” case.

Figures 1 to 3 (Examples 1 to 3). Note that all go up steeply for high percentile cases. Also note that the lowest value on the  $i$  axis is 0. This is because for  $i < 0$ , the KC indicates to avoid the bet. For example 3,  $i = 0$  until  $f \approx 0.38$ , because the KC indicates to avoid the bet. Otherwise I would find  $i < 0$  as I did for Figures 1 to 3.

#### 5. MEASURING $A$ , $B_0$ , $\pi$ , $\beta$ , $P_0$ USING THE DELPHI METHOD

The previous sections explain several of the key ideas that I want to convey in this article: (1) That to maximize the growth rate ( $i$ ) of a company, there is an optimal investment fraction ( $f$ ) for each project in an innovation portfolio. (2) This fraction depends on the probability of success ( $p$ ), the loss ratio ( $a$ ), and the win ratio ( $b$ ). And new to this article is (3) The values of  $b$  and  $p$  can increase with learning, for which I use a simple linear model according to eqs 7a and 7b. The previous analysis strongly suggests that  $\pi$  and  $\beta$  are essential metrics for the valuation and budgeting of the innovation function of a company.

In this section I propose a method to estimate the parameters for the Kelly strategy, including  $\pi$  and  $\beta$ . Whereas measuring the  $\pi$  and  $\beta$  parameters required for this article might seem very

difficult, since it would be hard to collect data with sufficient precision and accuracy to regress a model, I propose a much simpler process, based on the literature. I believe that companies who engage this process and the probabilistic thinking in this article will have a substantial advantage in innovation and therefore public service, as well as profitability; therefore the “expected value of the information”<sup>5</sup> will likely make it worthwhile to attain.

This section does not draw on directly related data that I or others have taken, but rather draws on a large body of literature concerning the Delphi forecasting method, and attaining “10–50–90 estimates” for agile estimation.<sup>4,6</sup> As Sam Savage states, a measurement is not a number, but a distribution.<sup>7</sup> We sometimes represent this distribution with a mean value and standard deviation, but in fact the distribution could be much more complicated in shape.

In that sense the method I propose is a hypothesis. Nevertheless I think it is very important to propose it here. The method uses a type of “wisdom of the crowds”, in which the members of the “crowd” have experience and expertise in the relevant innovation area, and so bring useful information to the process. The 10–50–90 refer to participants’ estimates of the 10th percentile answer (the “poor” outcome), the 50th percentile answer (the “median” outcome), and the 90th percentile answer (the “good” outcome).

The Delphi method is relatively fast and inexpensive, and so is used widely in software development for estimating the “velocity” of software development.<sup>8</sup> It works best with a small number of participants, perhaps 5 to 9, that cover a range of expertise about the situation. The method typically has 2–3 rounds alternating between a “scoring phase” and an “information phase”. The scoring phase is always anonymous, so that nobody “dominates the room”, and so that all voices (and therefore information) register. During the information phase, participants share risks and opportunities they see that justify their (anonymous) score, and this allows others in the room to access information that will enable them to update their score. Below are steps in a version of the Delphi method.<sup>9</sup> At the end of the section I provide a few extra comments.

### Preparation.

1. Prepare a spreadsheet to enter scores for 5–9 people, including comments and questions. An example is given in the Google Sheet link provided after Step 9 below, which the reader is free to copy and use. I’ve found that due to the large initial spread in participants’ entered values, it is helpful to average the log of scores, rather than the scores themselves. The sheet also calculates 90% confidence intervals for the parameters.
2. Assemble your Integrated Innovation Team (IIT). Have 5–9 members present. If too many, this small meeting can become harder to coordinate; if too few, you’ll lose reliability of the data. The meeting can be in person or by video, but the participants should only discuss the items below when called on. The scoring phase remains anonymous (i.e., “secret ballot”).
3. Describe a scenario in a 1-page narrative. You might briefly describe the customer and offering, the Innovation Team, some of the known risks, the plan and budget, leadership’s commitment level, and other information relevant to you. Let the IIT members read the page, and let them know that you realize that this exercise will seem somewhat vague, and that they should hold their

questions until the discussion portion, except for clarification questions.

### Scoring Phase.

4. State the question at the end, such as, “How many work days will it be to delivery, with today being time = 0?”
5. Give numerical values. Include a median time (50th percentile, in the middle), a fast time (90th, “good”, such that you would be faster in only 10% of possible cases), and a slow time (10th, “poor”, such that you would be slower in only 10% of possible cases). List a single number for each, since the range is given by the three numbers (poor-middle-good). Then, have the IIT list any questions each needs to have answered, in order to change their estimates up or down. Emphasize that all input will be discussed, but will also be anonymous.
6. Encourage the IIT to give responses. For some people this exercise seems simple, but some people absolutely hate to guess without full certainty, or knowing what others would think. Give the respondents enough time and encouragement, and then collect the results. The input could be on sheets of paper that are typed into the spreadsheet, or typed directly into the sheet.

### Information Phase.

7. Enter the scores into the spreadsheet and obtain the results for the parameters of interest:  $a$ ,  $b_0$ ,  $\beta$ ,  $p_0$ ,  $\pi$ , and time ( $t$ ). You’ll need to enter numerical values for each parameter. Also enter any risks and opportunities you see with each parameter, framed as a question. For example, “Is our learning ( $\pi$  and  $\beta$ ) going to be lower since leadership has not expressed much commitment?”
8. Share the scores and comments and questions with the full team. Minimize the discussion especially in the first round, to avoid groupthink and other biases.
9. Iterate back to Step 4, and repeat until changes are small enough for you.

I have made a sample spreadsheet that can be used to guide your Delphi process. It is a Google Sheet (<https://tinyurl.com/332azybs>) that can be used to collect data from up to 10 participants (A–J). To use the sheet you’ll need to make a copy for yourself so you have “Edit” permission. Each team member is given a letter, and they change only their letter (hopefully not peeking at other letters to avoid bias). The sheet collects the scores and comments, and gives the overall view of the Team in the “Delphi” tab. Because answers can vary widely, the sheet averages the log of the answers, effectively giving a geometric average.

The Delphi method has a history going back to the late 1950s with the RAND corporation. Delphi’s advantages over prediction markets have been discussed in the literature.<sup>10,11</sup> Experiments have shown advantages of Delphi over Face-to-Face meeting forecasts, and reveal that participants with the least conventional views tended to move their estimates in the right direction during the course of the meeting, although not always enough.<sup>9</sup> Comparison to other group forecasting methods reveals that Delphi for various types of groups (e.g., economists, MBA and other students, electronics engineers, medical personnel), and for various types of problems (e.g., banking, education, government, surgery), is usually superior in accuracy to other methods.<sup>12</sup>

Does the Delphi method qualify as a type of measurement? Can humans really be used as a measurement instrument to

produce accurate, precise, repeatable measurements? Hubbard<sup>4</sup> and Tetlock<sup>13</sup> both claim yes. Hubbard defines a “measurement” as “a set of observations that reduce uncertainty, where the result is expressed as a quantity.” The purpose is uncertainty reduction (in line with Sam Savage), not uncertainty elimination. He gives three reasons why anything (the title of his book) can be measured: (1) If it matters at all, it is detectable or observable. (2) If it is detectable, it can be detected as an amount, or a range of possible amounts. (3) If it can be detected as a range of possible amounts, it can be measured. Whether something *should* be measured is a separate question. Some quantities might not be measured for ethical reasons. Some quantities might not be measured because their expected value of perfect information (EVPI) is too low; that is, it is just not worth the cost to know.<sup>14</sup>

Importantly, the Delphi method and human measurement techniques benefit from calibration. To paraphrase Hubbard,<sup>4</sup> an important part of human measurement calibration is to get people to think that statistics is real, and that you can know *something* about a problem (i.e., bring some information about it), even without knowing *everything*. He claims that calibration training works on 85% of people within 5 rounds. Another book looks at “superforecasters”, which are people who have a natural tendency to make accurate forecasts, whether that is due to nature or nurture or both. This book claims that teams of superforecasters do *better* than individuals, not worse. The traits of superforecasters include that they are pragmatic, have a growth mindset,<sup>15</sup> are tenacious, are continually updating their view of the world in both knowledge and concept, and are good with numeracy (especially probability).

## 6. DISCUSSION OF BUDGETING AND SEQUENTIAL PROJECTS

My previous article (ref 1) discussed how to go from *fraction* ( $f$ ) to the optimal *expenditure* based on revenue ( $R$ ) and initial capital bankroll ( $W_0$ ), when the loss ratio  $a = 1$ :

$$E(a = 1) \approx pR \left( 1 - \frac{R}{W_0} + \frac{pR}{W_0} \right) \quad (18)$$

If you have all the capital in the world, that is,  $W_0 \rightarrow \infty$ , then taking the limit of eq 18 as  $W_0 \rightarrow \infty$  gives

$$E(W_0 \rightarrow \infty, a = 1) = pR \quad (19)$$

Once we have a value for  $E$  for each project in the portfolio, we can sum them to determine the optimal total size of innovation investment. A 2012 article by Knott used a Cobb-Douglas style production function to give the output ( $Y$ , revenue) in terms of the capital investment ( $K$ ), labor investment ( $L$ ), and R&D investment ( $R$ ).<sup>16</sup> The author developed a metric called Research Quotient (RQ), and ranked companies according to their RQ.<sup>17</sup> Based on the RQ, she determined whether a company should invest more or less in innovation. However, because the Kelly strategy provides a criterion that maximizes growth rate, the KC gives an alternative way to budget projects within an organization. Furthermore, with fairly accurate  $b$  and  $p$  data taken over time, the KC can give more precise values for budgeting than RQ, and can be improved over time as the human “measurement instruments” are calibrated to give better estimates.

In some cases, an overall project depends on two or more subprojects, where one must be completed before another can be started, and I’ve had companies ask me how to handle this. If a

project depends on the sequential success of two or more subprojects, the KC can be used to determine how much effort to put toward each part separately, versus in their integrated tandem. For example, if the overall success probability ( $p$ ) depends on two sequential projects, then  $p = p_1 p_2$ . If the value of  $f_{KC} > 0$  for the tandem, then that means that the highest growth will occur when the appropriate  $f_{KC}$  is put toward studying the joint nature of 1 and 2, rather than separately examining 1 and 2 and combining them later.

As stated in ref 1, we can also use the KC in a hierarchical manner. It can be used for corporate-level budgeting (e.g., should we allocate to R&D, better manufacturing, marketing, or M&A), subfunctional budgeting (e.g., within R&D, amount to budget toward analytical services, chemical synthesis, software development), the question level (i.e., of the 4 hypotheses to the question, the amount to budget to test each), and even the experimental level (e.g., for marketing, amount to invest in direct marketing, TV or radio, print).

If there is a portfolio of projects (bets)  $i = 1, 2, \dots, n$ , each with its own fraction ( $f_i$ ), loss ratio ( $a_i$ ), win ratio ( $b_i$ ), probability of success ( $p_i$ ), and rate of completion ( $k_i$ ), then we could write eq 3 more completely for  $m$  projects in a portfolio as

$$\mathfrak{R} = \frac{W}{W_0} = \prod_{i=1}^m (1 + f_i b_i)^{k_i p_i} (1 - f_i a_i)^{k_i (1-p_i)} \quad (20)$$

Furthermore, the  $a_i$ ,  $b_i$ ,  $p_i$ , and  $k_i$  could be written as functions of the  $f_i$  (e.g., as simple linear expressions as in this article, or more complicated). From an information processing perspective, eq 20 is the *fundamental equation of innovation*. It expresses the growth of wealth using just a few fundamental parameters ( $a$ 's,  $b$ 's,  $p$ 's,  $k$ 's), and suggests the philosophy that innovation is a process manufacturing operation that is largely about creating and shifting probabilities, along with win ratios and rates ( $k$ ), to maximize  $\mathfrak{R}$ . Ideally, max  $\mathfrak{R}$  is where the most profit resides, and the most service to society is delivered. Furthermore, the  $k$  and  $p$  parameters support the concept of “Intelligent Fast Failure” (IFF),<sup>18</sup> which recommends that since you will fail frequently during innovation, you should do it quickly, and learn as much as possible from each failure. That is, since  $p < 1$ , we expect to have “failed attempts”; however, we move through the failures at a fast rate (high  $k$ ), learning as much as we can to increase  $p$  and  $b$ , all to achieve max  $\mathfrak{R}$  and service.

The principles in this article can in fact be applied broadly in life or economics, for instance in analyzing the use of seatbelts in cars, cheating on exams or taxes, or doing fun activities that have a level of danger. Whenever the result depends on a serial set of mostly independent events, the KC or a variant is a useful way to examine the problem.

## 7. CONCLUSION

To maximize your growth rate ( $i$ ) and maintain a lower probability of “going bust” for innovation, you need to invest the proper amount in your projects. Investing either more or less will be suboptimal. A previous article (ref 1) showed that you can use the Kelly gambling (or investing) strategy to make the best decisions. To use the Kelly strategy, you must know the anticipated win ratio ( $b$ ) of an innovation project and its estimated probability ( $p$ ) of success.

In this article I recognize explicitly that both  $b$  and  $p$  will likely increase when you bet a fraction ( $f$ ) of your innovation budget on a project, due to learning that increases both  $b$  and  $p$ . This article analyzes how the optimal Kelly fraction  $f_{KC}$  changes when

learning takes place. I use linear approximations  $p = p_0 + \pi f$  and  $b = b_0 + \beta f$  to quantify the learning. As  $\pi$  and/or  $\beta$  increases, the optimal Kelly Criterion value  $f_{KC}$  increases, often significantly, also giving a dramatic increase in growth rate ( $i$ ).

Key ideas from this article are listed here:

1.  $f_{KC}$  and growth rate ( $i$ ) increase with learning. Learning means increasing your probability of success, or the value of your successes. The sheet given earlier can be used for the numerical calculation.
2. Measurements are an asset. A focus on measurements and the use of the Kelly algorithm will improve both the budgeting toward innovation (for maximum growth and lower risk) and the valuation of the innovation processes of a company.
3. Measurements can be taken using the Delphi method. To get more accurate values for  $f$  when learning occurs, it is recommended that organizations measure  $a$ ,  $b_0$ ,  $p_0$ ,  $\pi$ ,  $\beta$ , and  $k$  over time, where  $k$  is the rate of attempts made per time. For the measurements I propose the Delphi method as a relatively fast and inexpensive method, and give a template spreadsheet given previously.
4. Equation 20 expresses growth due to innovation, using parameters that can be monitored over time to improve the budgeting and valuation of a company's innovation processes.

I am not aware of any companies taking the data required for the model in this article. However, I propose that companies begin to take data that will yield the parameters  $a$ ,  $b_0$ ,  $p_0$ ,  $\pi$ ,  $\beta$ , and  $k$  over time. Historically it often happens that data are not taken until there is a theoretical framework calling for the data. This article provides the framework to use these parameters, enabling "probability processing" to improve the probability of success. I expect that having a measurement system to yield these parameters will be a great asset to an innovative company.

## AUTHOR INFORMATION

### Corresponding Author

Darrell Velegol – Penn State University, Department of Chemical Engineering, University Park, Pennsylvania 16802, United States; The Knowlecular Processes Company, State College, Pennsylvania 16803, United States; [orcid.org/0000-0002-9215-081X](https://orcid.org/0000-0002-9215-081X); Email: [darrell@knowlecular.com](mailto:darrell@knowlecular.com), [velegol@psu.edu](mailto:velegol@psu.edu)

Complete contact information is available at:  
<https://pubs.acs.org/10.1021/acs.iecr.2c03406>

### Notes

The author declares the following competing financial interest(s): Darrell Velegol is President of the Knowlecular Processes Company, which does consulting in innovation processes.

## ACKNOWLEDGMENTS

I thank Professor Kyle Bishop for many helpful discussions regarding the Kelly method and Venkat Venkatasubramanian for insights about entropy and other economic points.

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